Blade-Coating Equations for a Knife Blade and a Round Obstruction

J. A. TALLMADGE
and M. TUNTIBLARPHOL
Department of Chemical Engineering,
Drexel University,
Philadelphia, PA 19104

Film thickness in continuous, free coating of flat sheets and drums at high speeds are often reduced and controlled by knives and blades. In this paper, we present empirical equations to describe: (a) a round obstruction and (b) a wedge knife, based on the heretofore unpublished data of Tuntiblarphol (1970). These equations are also compared with published equations for long and for short flat obstructions.

The apparatus used is shown in Figure 1 (Tuntiblarphol and Tallmadge, 1970, 1971). Some of the independent parameters are coating speed u, gap opening d, gap length L, and fluid properties such as viscosity μ and density ρ. Three dependent coating parameters were measured: (1) mass flow rate entrained, w; (2) the small film thickness above the obstruction, h_A; and (3) the large film thickness below the obstruction, h_B. Both film thicknesses were those in the constant thickness regions, which are a distance from the menisci.

GRAVITY GAP EQUATIONS (FOR LONG GAPS)

For the case of a long, flat gap ($L/d \gg 1$), there are viscous and gravitational forces and one dimensional flow in the gap, or

$$\mu(d^2v/dy^2) - \rho g = 0 \tag{1}$$

Based on Eq. 1 and no slip at both walls in the obstruction, the predicted entrainment has been shown by Tuntiblarphol and Tallmadge (1970) to be:

$$w = (\rho gud/2)(1 - (\rho gd^2/6\mu u)) \tag{2}$$

$$(h/d)^3 - 3(h/d)(\mu u/\rho gd^2) + \tfrac{3}{2}(\mu u/\rho gd^2) - \tfrac{1}{4} = 0 \tag{3}$$

Equations 2 and 3 are called the "gravity gap" theories of obstructed flow, because gravitational forces are included in this simple theory. These equations may be expressed in nondimensional form using the Z number, where $Z \equiv u(\mu/\rho gd^2)$ and is the ratio of viscous to gravitational forces. For a given fluid and gap size, Z may be considered to be a nondimensional withdrawal speed. No inertial forces are implied by Z. (Some scholars may wish to describe Z as the ratio of Froude number to the Reynolds number.) The nondimensional forms of the dependent parameters (w and h) are $W \equiv w/\rho bud$ and $H \equiv h/d$. Thus, in nondimensional form, the gravity gap Eqs. 2 and 3 become

$$W = (\tfrac{1}{2})(1 - (\tfrac{1}{6}Z)) \tag{4}$$

$$H^3 - 3HZ + (\tfrac{3}{2})Z - (\tfrac{1}{4}) = 0 \tag{5}$$

Film-thickness Eqs. 3 and 5 have two positive roots which correspond to the film thickness above (h_A) and below (h_B) the obstruction.

The gravity-gap Eqs. 4 and 5 were verified experimentally using a 250 mN·s/m^2 liquid (250 cp), a 25 mm long gap, and gap size openings from 0.4 to 3.2 mm; thus, the long gap L/d ranged from 64 to 8 (Tuntiblarphol and Tallmadge, 1970).

The positive roots of thickness Eq. 5 may be written in explicit form, using the following functions of Z (based on Eq. 5):

$$A = (\tfrac{1}{2})[(\tfrac{3}{4})Z + \tfrac{1}{4}] \tag{5a}$$

$$B = (Z^3 - 0.563Z^2 + 0.187Z - 0.0153)^{1/2} \tag{5b}$$

$$\theta = \frac{1}{3}\tan^{-1}(B/A) \tag{5c}$$

Using A, B and θ, the solutions for upper thickness (H_A) and lower thickness (H_B) predicted by Eq. 5 are:

$$H_A = (A^2 + B^2)^{1/6}(\cos\theta - \sqrt{3}\sin\theta) \tag{5d}$$

$$H_B = (A^2 + B^2)^{1/6}(\cos\theta + \sqrt{3}\sin\theta) \tag{5e}$$

The roots and forms of the thickness equations are discussed in more detail in thesis form (Tuntiblarphol, 1970).

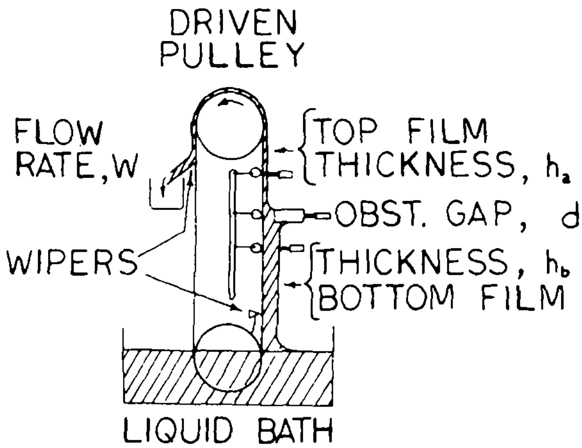

Figure 1. Schematic diagram of the blade coating apparatus.

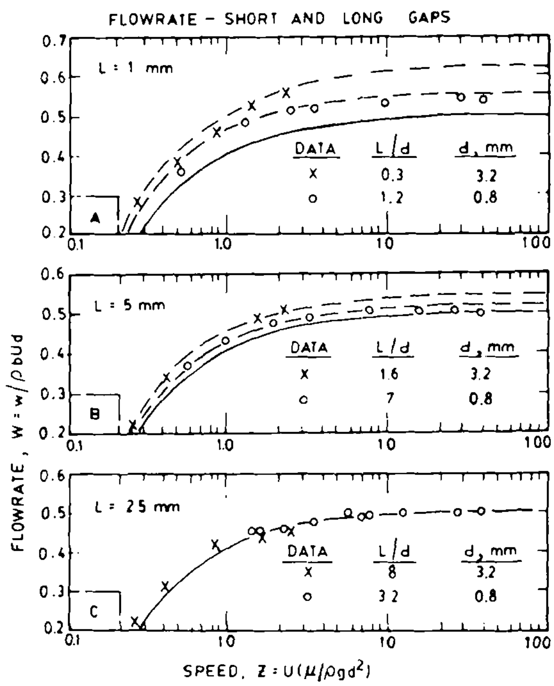

Figure 2. Flow rate data comparison of the short-gap Eq. 6 (dashed lines are for small L/d) (solid lines are Eq. 4 for large L/d). (Viscosity 240 mN·s/m^2).

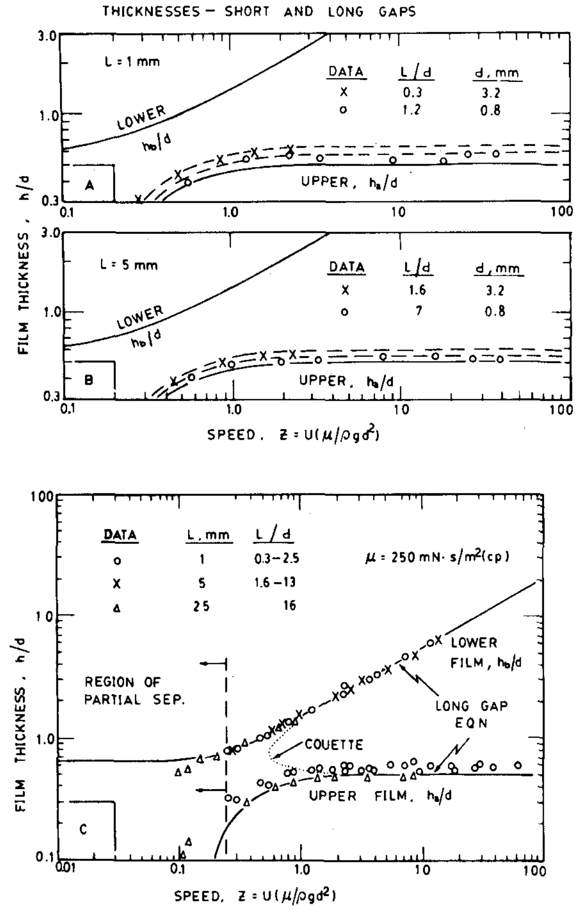

Figure 3. Thickness data comparison of short-gap Eq. 7 (dashed lines are for small L/d) (solid lines are Eq. 5 for large L/d) (dotted line is for the Couette gap Eq. 5s).

COUETTE FLOW (A SPECIAL CASE OF THE LONG GAP)

At large Z (high speeds), the effect of gravity becomes negligible. Thus Eqs. 4 and 5 reduce to the "Couette gap" theories for flow (W_c) and thickness (H_c), which have been shown to be

$$W_C = 1/2 \tag{4s}$$

$$H_C^3 - 3H_C Z + (3/2)Z = 0. \tag{5s}$$

Thus gravity becomes influential at low Z. Based on a 5% deviation criterion (for the mass flowrate W of Eq. 4 from 1/2), gravity becomes influential at Z below 3.

SHORT GAPS

Experimental studies with short gaps (including L/d of 1.6, 1.2 and 0.3) led to the following empirical correlations of Tuntiblarphol and Tallmadge (1971); these equations describe the short gap flow rate (W_s) and short gap top thickness (H_{AS}) in terms of the long gap values (W, H_A), as follows:

$$W_s/W = 1 + 0.115(d/L)^{2/3} \tag{6}$$

$$H_{AS}/H_A = 1 + 0.190(d/L)^{1/3} \tag{7}$$

Parts A and B of Figure 2 indicate the agreement of the mass flow rate (Eq. 6) with short gap data. Part C of Figure 2 shows the agreement with the long gap model (solid line), not only at high speeds (high Z), but also for low speeds (low Z below 3), where gravity is important.

Figure 3 shows the equivalent agreement with both upper and lower film thicknesses. Also shown in Part C (of Figure 3) is that partial separation of the film from the gap was visually observed

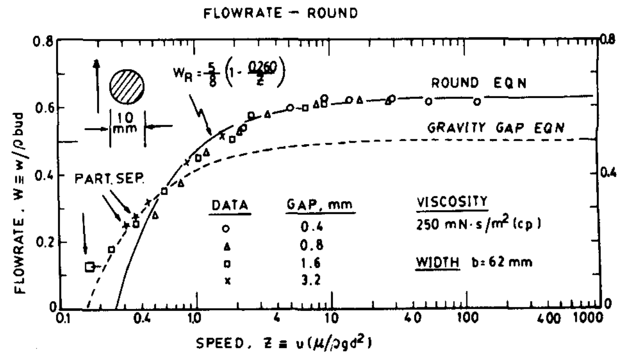

Figure 4. Flow rate data comparison of round knife Eq. 10.

to occur at low Z below 0.2. Here surface tension effects begin to be appreciable and low-speed operability limits occur because of instabilities and surface effects. It is somewhat surprising that these equations remain valid below Z of 0.5 (where the lower thickness H_B becomes less than 1); this probably occurs because the experiments were done at decreasing speeds so that contact with the gap surface was maintained by the continued contact due to meniscus effects.

ROUND OBSTRUCTION STUDY

A long, horizontal cylinder of 10.0 mm diameter was placed at four gap locations from the belt (0.4, 0.8, 1.6, and 3.2 mm). Study of the flowrate and thickness data for the 0.8 mm gap indicated the data lay between the long-gap, gravity theory for 0.8 and 1.6 mm gaps. With similar data for other gaps, it was therefore decided to replace the gap size "d" in theoretical Eqs. 4 and 5 by an empirical ratio *md*. Thus the empirically adjusted equations for flow (W_R) and thickness (H_R) for this round case are:

$$W_R = (m/2)(1 - (m^2/6Z)) \tag{8}$$

$$H_R^3 - 3H_R Z + 3mZ/2 - m^3/4 = 0 \tag{9}$$

At large speeds (large Z), H_R was found to approach 5/8 (whereas long gap H_L approaches 1/2). Therefore, $m = H_R/H_L = (5/8) \div (1/2)$, so that m was taken as 5/4. Using this empirical value of m in Eqs. 8 and 9, we obtain the two semiempirical equations for a round obstruction (or cylindrical knife), namely Eqs. 10 and 11:

$$W_R = (5/8)(1 - (25/96Z)) = 0.625(1 - 0.260/Z) \tag{10}$$

$$H_R^3 - 3H_R Z + 1.88Z - 0.49 = 0 \tag{11}$$

These expressions, as shown in Figures 4 and 5, compare very favorably with data. In order to obtain good agreement in the low speed region (of Z below 3), it was necessary to use the gravity gap equations rather than the Couette gap equations when selecting

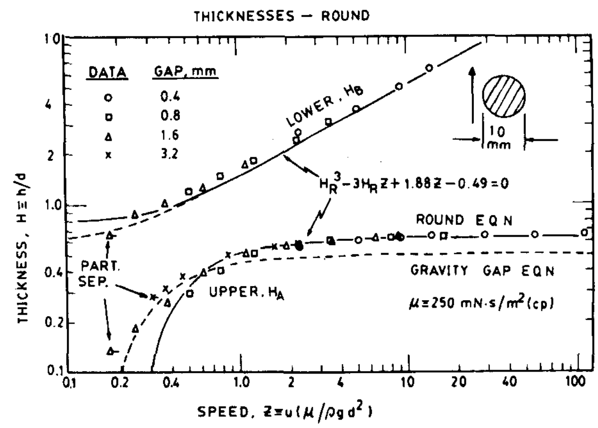

Figure 5. Thickness data comparison of round knife Eq. 11.

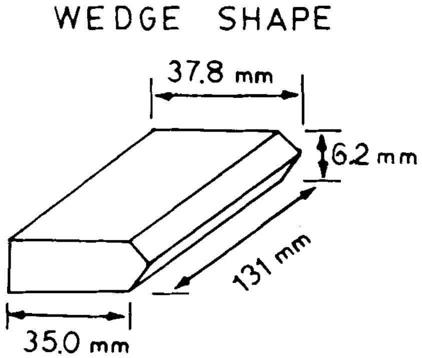

Figure 6. Dimensions of the wedge knife.

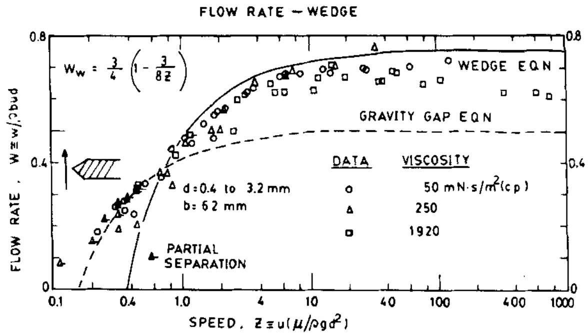

Figure 8. Flow rate data comparison of wedge knife Eq. 12.

a basis for the extended equations. For example, note that the minimum thickness H of about 0.7 predicted by the Couette gap equations occurs at a Z of about 0.5, as shown in Figure 3C. Thus the Couette gap equations are not suitable or valid at low Z.

WEDGE STUDY

For a wedge-shaped knife, the angle and thickness of the wedge may play important roles. The aluminum wedge used here is shown in Figure 6. This wedge had a vertical thickness of 6.2 mm and a point which protruded out by 2.8 mm from the edge. Thus the length of the aluminum block varied from 35.0 mm at the side edge to 37.8 mm at the center point. The width of the block was 131 mm, so that it was far wider than the moving belt, which was 62 mm wide.

Using a similar empirical analysis to that used with the round obstruction, the m value for the wedge was found to be $3/2$. Thus the semiempirical expressions for a wedge knife flow (W_w) and thickness (H_w), obtained from extension Eqs. 8 and 9, are given by Equations 12 and 13:

$$W_w = (3/4)(1 - (3/8Z)) = 0.75(1 - 0.375/Z) \quad (12)$$

$$H_w^3 - 3H_wZ + 2.25Z - 0.84 = 0 \quad (13)$$

Comparison of these semiempirical expressions with data is shown in Figures 7 and 8. The agreement with film thickness H is very good. Except for some deviation with the highest viscosity oil, agreement with flow rate W is also good. As with the expressions for the round obstructions, these semiempirical equations for the wedge knife were obtained by empirical extensions of the hydrodynamic theory given in Eqs. 1 to 5.

COMMENT

The expressions for the gravity gap model have been extended using data, to both a wedge-shaped knife and a round knife, as used in blade coating. (Tallmadge, 1982). The nondimensional speed used here was

$$Z = u(\mu/\rho g d^2) \quad (14)$$

A more general expression for Z is Z′, which includes the effect of pressure drops across the gap, and is based on an earlier derivation by Tuntiblarphol and Tallmadge (1970). Here Z′ is:

$$Z' \equiv \frac{\mu u}{a d^2} \equiv \frac{\mu u}{d^2(\rho g + (\Delta P/L))} \quad (15)$$

All the gap equations given above may be written in terms of Z′, in place of Z. One unanswered question involves the accuracy of knife Eqs. 10 to 13 using the Z′ of Eq. 15 to describe cases of pressure drops present. (Tallmadge, 1982).

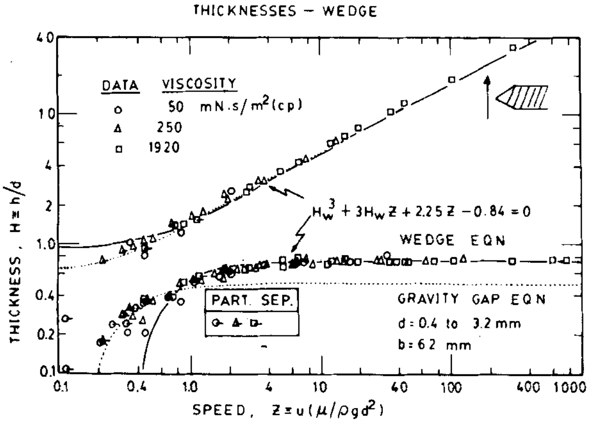

Figure 7. Thickness data comparison of wedge knife Eq. 13.

ACKNOWLEDGMENT

The authors are grateful to Eastman Kodak for partial support of this work.

NOTATION

- a = parameter, given by Eq. 15
- A = function of Z, Eq. 5a
- b = film width, mm
- B = function of Z, Eq. 5b
- d = gap opening, mm
- g = gravity, mm/s²
- h = film thickness, constant region, mm
- H = nondimensional thickness, h/d
- L = gap length, vertical, mm
- u = solid speed or belt speed, mm/s
- v = liquid velocity in gap, Eq. 1, mm/s
- w = mass flow rate, g/s
- W = nondimensional flow rate, w/ρ bud
- y = distance from blade, mm
- Z = nondimensional speed, $u\mu/\rho g d^2$
- Z' = nondimensional speed, Eq. 15

Subscripts (on *W* and *H*)

- A = upper thickness (above gap)
- B = lower thickness (below gap)
- C = Couett gap theory prediction
- L = long gravity gap thickness
- R = round gap, Eqs. 10 and 11
- S = short gap, Eqs. 6 and 7
- W = wedge knife, Eqs. 12 and 13

Greek Letters

- ΔP = pressure drop across gap, Eq. 15
- μ = liquid viscosity, mN·s/m²
- ρ = liquid density, kg/m³
- θ = function of Z, Eq. 5c

LITERATURE CITED

Tallmadge, J. A., "Blade Coating—Comparison of Data with Theory," Orlando Winter National AIChE Mtg. (March, 1982).

Tuntiblarphol, M., "Obstructed Flow of Newtonian Liquids on Flat Plate Withdrawal," M.S. Thesis in Chem. Engr., Drexel Univ. (1970).

Tuntiblarphol, M., and J. A. Tallmadge, "Coating by Obstructed Flow in Plate Withdrawal," *Can. J. Chem. Engr.*, 48, p. 617 (1970).

Tuntiblarphol, M., and J. A. Tallmadge, "Length Effects in Short, Flat Withdrawal Obstructions," *Ind. Engr. Chem. Process Design and Dev.*, **10**, p. 353 (1971).

Manuscript received February 23, 1982; revision received and accepted August 26, 1982.

Simple Models for Nonideal Vapor-Liquid Equilibrium Calculations

SANDRO MACCHIETTO
E. H. CHIMOWITZ
and T. F. ANDERSON

Department of Chemical Engineering
The University of Connecticut
Storrs, CT 06268

In process simulation and design the rigorous thermodynamic functions which model vapor-liquid equilibria (VLE) are evaluated many times and can account for up to 80% of the total computation costs (Westerberg et al., 1979). This computational effort can be substantially reduced by using simplified local models to represent the rigorous thermodynamic properties. Models of this type have been presented by Leesley and Heyen (1977), Boston and Britt (1978), and Barret and Walsh (1979). A new approach for process design calculations involving the use of local models was detailed by Chimowitz et al. (1982 a,b). Here, our purpose is to present some new insights into the properties of the local models and propose areas of process design which stand to gain from the use of the local model concept.

SIMPLIFIED VLE MODELS

Vapor-liquid equilibrium in a mixture of n components if often defined in terms of the equilibrium ratio, K, defined by:

$$K_i = y_i/x_i = (\gamma_i f_i^o)/(\phi_i P) \qquad (i = 1 \ldots n) \qquad (1)$$

where γ_i is the activity coefficient, f_i^o is the standard-state liquid fugacity, ϕ_i is the vapor fugacity coefficient, and P is the system pressure. In general, activity and fugacity coefficients are nonlinear functions of temperature, pressure and composition, and Eq. 1 cannot be solved explicitly for any of these variables.

A number of simpler expressions may be derived from Eq. 1 by making various assumptions regarding the nonideality of the solution. For example, we may assume that the relative volatility α_i is constant (α_i is defined as K_i/K_r, with r indicating a reference component), or that K_i is a function of temperature only. We may also take into account the major composition effects, but with only a very approximate functional dependence.

Representative models based on these considerations are shown in Table 1 (Chimowitz, 1982). The presence of adjustable parameters in the model equations makes it possible to fit rigorous VLE data about a particular point or region of interest in the two-phase envelope. Thus, it is quite possible for a simple model to represent the rigorous thermodynamic K_i well, at least locally, even if some of the assumptions used to derive it may be considered very drastic.

TABLE 1. REPRESENTATIVE LOCAL K-VALUE MODELS

Model	Defining Equations	Model Type	Min. # of Data
1	$ln\,(K_i) = a_i + b_i\,ln\,f_i^o - ln\,P$	Temperature only. Same as Leesley and Heyen's (1977) at low pressure	2
2	$ln\,(K_i) = a_i + b_i x_i + ln\,f_i^o - ln\,P$	All composition dependencies lumped into one term (Chimowitz et al., 1982a)	2
3	$ln\,(K_r) = a_r\,ln\,f_r^o + b_r(1 - x_r)^2 + c_r - ln\,P$ $ln\,(K_i/K_r) = a_i/T + b_i(1 - x_r)^2 + c_i(1 - x_i)^2 + d_i$	Relative volatility model respect to reference component (Chimowitz et al., 1982a)	4

Sandro Macchietto is presently at Dept. of Chemical Engineering and Chem. Tech., Imperial College, London SW7 2BY, England, and E. H. Chimowitz is currently at the Department of Chemical Engineering, University of Rochester, Rochester, NY 14627.

PERFORMANCE OF LOCAL MODELS

The capabilities of the local models shown in Table 1 are most conveniently assessed by using them in bubble-point calculations. Resulting temperatures and compositions are compared to those obtained using rigorous thermodynamic subroutines (Prausnitz et al., 1980). The corresponding errors provide a measure of the performance of the models.

In making these comparisons, we distinguish between the ability of the models (a) to interpolate accurately in the range where the model was fit and (b) to be extrapolated outside this range. Both capabilities are important in process design and simulation calculations.

To illustrate, we first consider the binary mixtures shown in Table 2. For each mixture at an arbitrary composition of 50 mol %, rigorous equilibrium ratios were generated at the bubble point and dew point. The local models were fit to these rigorous data. Model 3 required two additional rigorous data points (calculated at intermediate conditions) to obtain its parameters. The models and their associated parameters were then used to compute the equilibrium curve throughout the entire composition range.

Table 2 compares temperature and vapor compositions predicted by the local models with the corresponding rigorous values. In general, greater accuracy is achieved as a more complex model is used. However, errors are usually within 1% for all models when limited to the region in which they were fit. Mixture A represents an exception. This binary has a wide composition range from the